

# Scaling Functions and Superscaling in Medium and Heavy Nuclei

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The scaling function  $f(\psi')$  for medium and heavy nuclei with  $Z \neq N$  for which the proton and neutron densities are not similar is constructed within the coherent density fluctuation model (CDFM) as a sum of the proton and neutron scaling functions. The latter are calculated in the cases of  $^{62}\text{Ni}$ ,  $^{82}\text{Kr}$ ,  $^{118}\text{Sn}$ , and  $^{197}\text{Au}$  nuclei on the basis of the corresponding proton and neutron density distributions which are obtained in deformed self-consistent mean-field Skyrme HF+BCS method. The results are in a reasonable agreement with the empirical data from the inclusive electron scattering from nuclei showing superscaling for negative values of  $\psi'$ , including those smaller than  $-1$ . This is an improvement over the relativistic Fermi gas (RFG) model predictions where  $f(\psi')$  becomes abruptly zero for  $\psi' \leq -1$ . It is also an improvement over the CDFM calculations made in the past for nuclei with  $Z \neq N$  assuming that the neutron density is equal to the proton one and using only the phenomenological charge density.

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The studies of the scaling phenomenon which has been observed in inclusive electron scattering from nuclei make it possible to gain information about basic nuclear characteristics such as the local density  $\rho(r)$  and momentum distribution  $n(k)$  in nuclei. This concerns firstly the  $y$ -scaling (e.g. [1, 2, 3, 4, 5]). It was found also that a properly defined function  $f(\psi')$  of another scaling variable (the  $\psi'$ -variable) has a superscaling behavior. The latter means that for  $\psi' < 0$  this function is independent on the transfer momentum  $q$  (at  $q > 500$  MeV/c) and on the mass number for a wide range of nuclei from  $^4\text{He}$  to  $^{197}\text{Au}$ . This was firstly considered within the framework of the RFG model (e.g. [6, 7, 8, 9, 10]). As pointed out in [8], however, the actual nuclear dynamical content of the superscaling is more complex than that provided by the RFG model. It was observed that the experimental data have a superscaling behavior for large negative values of  $\psi'$  (up to  $\psi' \approx -2$ ), while the predictions of the RFG model are for  $f(\psi') = 0$  at  $\psi' \leq -1$ . This imposes the consideration of the superscaling in realistic finite systems. Such works were performed [11, 12] in the CDFM [13, 14, 15], which is related to the  $\delta$ -function limit of the generator-coordinate method [11, 16]. The calculated CDFM scaling function  $f(\psi')$  agrees with the available experimental data from the inclusive electron scattering for  $^4\text{He}$ ,  $^{12}\text{C}$ ,  $^{27}\text{Al}$ ,  $^{56}\text{Fe}$  and, approximately, for  $^{197}\text{Au}$  for various values of the transfer momentum  $q = 500, 1000, 1650$  MeV/c [11] and  $1560$  MeV/c [12], showing superscaling for negative values of  $\psi'$  including also those smaller than  $-1$  (in contrast to the RFG model result). It was shown in [11, 12] that the superscaling in nuclei can be explained quantitatively on the basis of the similar behavior of the high-momentum components of the nucleon momentum distributions in light, medium and heavy nuclei. It is known that the latter

is related to the effects of the short-range and tensor nucleon-nucleon correlations in nuclei (see, e.g. [13]). Our scaling function was obtained starting from that in the RFG model [6, 7, 8] in two equivalent ways, on the basis of the local density distribution and of the nucleon momentum distribution. This gives a good opportunity to study simultaneously the role of the nucleon-nucleon correlations included in  $\rho(r)$  and  $n(k)$  in the case of the superscaling phenomenon.

Here we would like to emphasize, however, that in [11, 12] we encountered some difficulties to describe within the CDFM the superscaling in the case of  $^{197}\text{Au}$  which was the most heavy nucleus considered. We related this to the particular  $A$ -dependence of  $n(k)$  in the model that does not lead to realistic high-momentum components of the momentum distribution in the heaviest nuclei. We followed in [11, 12] a somewhat artificial way to “improve” the high-momentum tail of  $n(k)$  in  $^{197}\text{Au}$  by taking the value of the diffuseness parameter  $b$  in the Fermi-type charge density distribution of this nucleus to be  $b = 1$  fm instead of the value  $b = 0.449$  fm (as obtained from electron elastic scattering experiments, see [17]). In such a case the high-momentum tail of  $n(k)$  for  $^{197}\text{Au}$  in CDFM becomes similar to those of  $^4\text{He}$ ,  $^{12}\text{C}$ ,  $^{27}\text{Al}$ , and  $^{56}\text{Fe}$  nuclei and this leads to a good agreement of the scaling function  $f(\psi')$  with the data also for  $^{197}\text{Au}$ . Discussing this in [11] we pointed out, however, that all the nucleons may contribute to  $f(\psi')$  for the transverse electron scattering and this could reflect on the diffuseness of the matter density for a nucleus like  $^{197}\text{Au}$  whose value can be different from that of the charge density used in our previous works [11, 12].

The aim of the present work is to apply the CDFM by using both proton and neutron densities for medium and heavy nuclei (for which  $Z \neq N$ ) in contrast to our

previous approach, in which we assumed that the neutron density was equal to that of protons and we used only the phenomenological charge density [17]. In our work now the total scaling function  $f(\psi')$  will be a sum of two scaling functions, those for protons and neutrons.

In [12] the CDFM scaling function  $f(\psi')$  was given in two equivalent ways, firstly, by means of the density distribution

$$f(\psi') = \int_0^{\alpha/(k_F|\psi'|)} dR |F(R)|^2 f_{RFG}(R, \psi'), \quad (1)$$

where

$$|F(R)|^2 = -\frac{1}{\rho_0(R)} \left. \frac{d\rho(r)}{dr} \right|_{r=R}, \quad (2)$$

$$\rho_0(R) = \frac{3A}{4\pi R^3}, \quad \alpha = \left( \frac{9\pi A}{8} \right)^{1/3} \simeq 1.52A^{1/3}, \quad (3)$$

$$f_{RFG}(R, \psi') = \frac{3}{4} \left[ 1 - \left( \frac{k_F R |\psi'|}{\alpha} \right)^2 \right] \left\{ 1 + \left( \frac{R m_N}{\alpha} \right)^2 \times \left( \frac{k_F R |\psi'|}{\alpha} \right)^2 \left[ 2 + \left( \frac{\alpha}{R m_N} \right)^2 - 2 \sqrt{1 + \left( \frac{\alpha}{R m_N} \right)^2} \right] \right\}, \quad (4)$$

( $m_N$  being the nucleon mass), and secondly, by means of the momentum distribution

$$f(\psi') = \int_{k_F|\psi'|}^{\infty} d\bar{k}_F |G(\bar{k}_F)|^2 f_{RFG}(\bar{k}_F, \psi'), \quad (5)$$

where

$$|G(\bar{k}_F)|^2 = -\frac{1}{n_0(\bar{k}_F)} \left. \frac{dn(p)}{dp} \right|_{p=\bar{k}_F} \quad (6)$$

and

$$n_0(\bar{k}_F) = \frac{3A}{4\pi \bar{k}_F^3}. \quad (7)$$

In Eq. (5) the RFG scaling function  $f_{RFG}(\bar{k}_F, \psi')$  can be obtained from  $f_{RFG}(R, \psi')$  (Eq. (4)) by changing there  $\alpha/R$  by  $\bar{k}_F$ . In Eqs. (1), (4), and (5) the Fermi momentum  $k_F$  is not a free fitting parameter for different nuclei as in the RFG model, but it is calculated in the CDFM for each nucleus using the corresponding expressions:

$$\begin{aligned} k_F &= \int_0^{\infty} dR k_F(R) |F(R)|^2 = \int_0^{\infty} dR \frac{\alpha}{R} |F(R)|^2 = \\ &= \frac{4\pi(9\pi)^{1/3}}{3A^{2/3}} \int_0^{\infty} dR \rho(R) R \end{aligned} \quad (8)$$

when

$$\lim_{R \rightarrow \infty} [\rho(R) R^2] = 0 \quad (9)$$

is fulfilled and

$$k_F = \frac{16\pi}{3A} \int_0^{\infty} d\bar{k}_F n(\bar{k}_F) \bar{k}_F^3 \quad (10)$$

when

$$\lim_{\bar{k}_F \rightarrow \infty} [n(\bar{k}_F) \bar{k}_F^4] = 0 \quad (11)$$

is fulfilled.

In [11, 12] we used the charge density distributions to determine the weight function  $|F(R)|^2$  in calculations of  $f(\psi')$  from Eqs. (1)–(4) and (8). In the present work we assume that the reason why the CDFM does not work properly in the case of  $^{197}\text{Au}$  is that we use in [11, 12] only the charge density, while this nucleus has many more neutrons than protons ( $N = 118$  and  $Z = 79$ ), and therefore proton and neutron densities may differ considerably. In this case the proton  $f_p(\psi')$  and neutron  $f_n(\psi')$  scaling functions will be given by the contributions of the proton and neutron densities  $\rho_p(r)$  and  $\rho_n(r)$ , correspondingly:

$$f_{p(n)}(\psi') = \int_0^{\alpha_{p(n)}/(k_F^{p(n)}|\psi'|)} dR |F_{p(n)}(R)|^2 f_{RFG}^{p(n)}(R, \psi'), \quad (12)$$

where the proton and neutron weight functions are obtained from the corresponding proton and neutron densities

$$|F_{p(n)}(R)|^2 = -\frac{4\pi R^3}{3Z(N)} \left. \frac{d\rho_{p(n)}(r)}{dr} \right|_{r=R}, \quad (13)$$

$$\alpha_{p(n)} = \left( \frac{9\pi Z(N)}{4} \right)^{1/3}, \quad (14)$$

$$\int_0^{\infty} \rho_{p(n)}(\vec{r}) d\vec{r} = Z(N), \quad (15)$$

and the Fermi momentum for the protons and neutrons is given by

$$k_F^{p(n)} = \alpha_{p(n)} \int_0^{\infty} dR \frac{1}{R} |F_{p(n)}(R)|^2. \quad (16)$$

The RFG proton and neutron scaling functions  $f_{RFG}^{p(n)}(R, \psi')$  have the form of Eq. (4), where  $\alpha$  and  $k_F$  are changed by  $\alpha_{p(n)}$  from Eq. (14) and  $k_F^{p(n)}$  from Eq. (16),

correspondingly. The normalizations of the functions are as follows:

$$\int_0^\infty |F_{p(n)}(R)|^2 dR = 1, \quad (17)$$

$$\int_{-\infty}^\infty f_{p(n)}(\psi') d\psi' = 1. \quad (18)$$

Then the total scaling function can be expressed by means of both proton and neutron scaling functions:

$$f(\psi') = \frac{1}{A} (Z f_p(\psi') + N f_n(\psi')) \quad (19)$$

and is normalized to unity.

The same consideration can be performed equivalently on the basis of the nucleon momentum distributions for protons  $n^p(k)$  and  $n^n(k)$  presenting  $f(\psi')$  by the sum of proton and neutron scaling functions (19) calculated similarly to Eqs. (12)–(19) (and to Eqs. (5), (6), (10), and (11)):

$$f_{p(n)}(\psi') = \int_{k_F^{p(n)}|\psi'|}^\infty d\bar{k}_F |G_{p(n)}(\bar{k}_F)|^2 f_{RFG}^{p(n)}(\bar{k}_F, \psi'), \quad (20)$$

where

$$|G_{p(n)}(\bar{k}_F)|^2 = -\frac{4\pi\bar{k}_F^3}{3Z(N)} \left. \frac{dn^{p(n)}(p)}{dp} \right|_{p=\bar{k}_F} \quad (21)$$

with  $f_{RFG}^{p(n)}(\bar{k}_F, \psi')$  containing  $\alpha_{p(n)}$  from Eq. (14) and  $k_F^{p(n)}$  calculated by

$$k_F^{p(n)} = \int_0^\infty d\bar{k}_F \bar{k}_F |G_{p(n)}(\bar{k}_F)|^2. \quad (22)$$

We calculate the scaling function for several examples, for the medium stable nuclei  $^{62}\text{Ni}$  and  $^{82}\text{Kr}$  and for the heavy nuclei  $^{118}\text{Sn}$  and  $^{197}\text{Au}$  following Eqs. (12)–(19) using the corresponding proton and neutron densities obtained in deformed self-consistent mean-field (HF+BCS) calculations with density-dependent Skyrme effective interaction (SG2) using a large harmonic-oscillator basis with 11 major shells [18, 19].

The results of the calculations of  $f(\psi')$  for  $^{62}\text{Ni}$ ,  $^{82}\text{Kr}$ ,  $^{118}\text{Sn}$ , and  $^{197}\text{Au}$  for  $q = 1000$  MeV/c are given in Fig. 1 and are compared with the experimental data (presented by a gray area and taken from [8]) obtained for  $^4\text{He}$ ,  $^{12}\text{C}$ ,  $^{27}\text{Al}$ ,  $^{56}\text{Fe}$ , and  $^{197}\text{Au}$ . The scaling functions are in a reasonable agreement with the data, which was not the case for  $^{197}\text{Au}$  calculated in [11] by using the experimental Fermi-type charge density only with parameter values  $b = 0.449$  fm and  $R = 6.419$  fm from [17] (see also Fig. 2 of Ref. [11]). It must be stressed that the theoretical proton densities used in the present calculations are close to

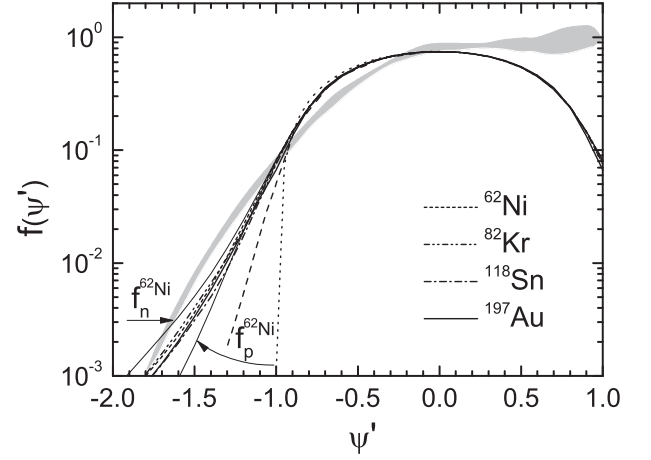


FIG. 1: Scaling function  $f(\psi')$  calculated in the CDFM for  $^{62}\text{Ni}$ ,  $^{82}\text{Kr}$ ,  $^{118}\text{Sn}$ , and  $^{197}\text{Au}$ . The results are obtained using Eqs. (12)–(19). The experimental data for  $^4\text{He}$ ,  $^{12}\text{C}$ ,  $^{27}\text{Al}$ ,  $^{56}\text{Fe}$ , and  $^{197}\text{Au}$  at  $q = 1000$  MeV/c taken from [8] are shown by the shaded area. The RFG result is presented by dotted line. The results of the calculations for  $^{197}\text{Au}$  [11] by means of Eqs. (1)–(4) using the Fermi-type charge density [17] are shown by dashed line.

the experimental charge densities. At the same time we note also the improvement in comparison with the RFG model result in which  $f(\psi') = 0$  for  $\psi' \leq -1$ . As an example the proton and neutron scaling functions for  $^{62}\text{Ni}$  are also given in Fig. 1, which clearly illustrates the different tails of the proton and neutron scaling functions when  $Z \neq N$ , as well as their role in building up the observed scaling function.

In conclusion, we point out that the scaling function  $f(\psi')$  for nuclei with  $Z \neq N$  for which the proton and neutron densities are not similar has to be expressed by the sum of the proton and neutron scaling functions. The latter can be calculated within the CDFM on the basis of the knowledge (obtained theoretically and/or experimentally) of the corresponding proton and neutron local density distributions or momentum distributions. We should also point out that the agreement with experiment is quite reasonable given that no adjustable parameter at all has been used in the present calculations.

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